Noname manuscript No.
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A Study on Dexterous Grasps via Parallel Manipulation Analogy

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Received: date / Accepted: date

Abstract This paper proposes a new approach for studying the dexterous grasping mechanisms via parallel manipulation analogy. The approach exploits the theories already developed for the dexterous robotic hands and the parallel manipulators. It also proposes an innovative conceptual design algorithm for dexterous grasping mechanisms with desired "dexterity" characteristics: mobility, connectivity, overconstraint, and redundancy. The provided quick mobility calculation formula is valid for all the grasping mechanisms whereas the other quick mobility calculation formulas are not. The proposed conceptual design algorithm is supported by example syntheses of a 3 dof translational motion dexterous grasping mechanism, a 3 dof (2 translational and 1 rotational) planar motion dexterous grasping mechanism and a 6 dof (3 translational and 3 rotational) spatial motion dexterous grasping mechanism.

Keywords Dexterous grasps · mechanism.

1 Introduction

Dexterous grasping mechanisms are rarely studied in the literature from the parallel manipulation point of view [1], and yet to be given a complete conceptual design method for their "dexterity" characteristics. Whereas a grasp (e.g., hand-plus-object closed-loop system) has huge similarities to the well studied parallel manipulators.

In this paper we study the dexterous grasping mechanisms by unifying the theories of parallel manipulators and the dexterous robotic hands. We

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also propose an innovative conceptual design algorithm for dexterous grasping mechanisms which takes into account the "dexterity" characteristics of the grasping mechanism:

Mobility: The number of independent coordinates required to define the configuration of a grasping mechanism (*e.g.*, palm-fingers-object closed-loop system. It is also called *degrees of freedom.*

Connectivity: The number of independent displacements (finite, infinitesimal) allowed between the two links of a grasping mechanism (*e.g.*, connectivity of fingers located between the palm and the object).

Overconstraint: The difference between the total number of coordinates that could loose their independence before forming a closed-loop mechanism (*e.g.*, palm-fingers system without the object) and the number of dependent coordinates after forming a closed-loop mechanism (*e.g.*, palm-fingers system with the object).

Redundancy: The difference between the mobility of the closed-loop grasping mechanism (*e.g.*, palm-fingers-object closed-loop system) and the connectivity of its grasped object with respect to base of the mechanism.

The seminal works on the dexterous robotic hands can be found in [2], [3], [4], [5], [6]. Salisbury in [2], [3] used Grübler's mobility formula and synthesized a three-fingered dexterous robotic hand with frictional point contacts yielding 6 degrees-of-freedom. Later, Tischler et. al. [4] proposed a new Melbourne method to synthesize complete list of kinematic chains for dexterous robotic hands. They showed that their method produces the least number of isomorphic chains in the complete list compared to other existing methods. This method was based on Freudenstein's mobility formula [7] and screw system theory. Tischler et al. [5] showed how to use this Melbourne method to synthesize fingers of a robotic hand by taking into account the kinematic constraints imposed by the grasping and manipulation of an arbitrary object. Fingers, that they synthesized, consider only frictional point contacts and contain one dof joints. Then, Lee and Tsai in [6] used Tsai's mobility formula to synthesize n-fingered dexterous robotic hands ($3 \leqslant n \leqslant 7$) with different types of contact yielding an atlas of feasible multi-fingered robotic hand structures with mobilities three to six. Fingers, that they synthesized, also contain only one dof joints.

The main drawback of these aforementioned methods is that their mobility calculation formulas are not valid for all kind of mechanisms, and thus the proposed set of feasible grasping mechanisms are incomplete and may contain erroneous solutions. Recently in [8], Gogu reformulated properly the quick mobility calculation formula, which is valid for all mechanisms incorporated in parallel manipulators with $n \geqslant 2$ limbs. Gogu also gave formulas for the connectivity, overconstraint, and redundancy of a mechanism. Here we adopt and adapt these formulas to use them for the first time in dexterous grasping mechanism synthesis. We remark that this study improves authors' previous paper [9] first by presenting it from a parallel manipulation point of view, second by extending its content with the evolutionary morphology (EM) algorithm and third by providing a new conceptual design whose structure cannot be synthesized by the other existing design methods. This study thus contributes for the dexterous grasping mechanisms in three ways:

1. It proposes an innovative design algorithm which takes into account all the essential "dexterity" characteristics of the grasping mechanism: mobility, connectivity, overconstraint, and redundancy. This has never been addressed before.
2. It allows to fix the bugs of other dexterous grasping mechanisms design methods with the correct mobility formula of mechanisms.
3. It presents new dexterous mechanisms for grasping (see figures 2, 3 and 4).

2 Compared to and with Parallel Manipulators

To: A parallel manipulator has a closed-loop topology, so does a grasping mechanism with the grasped object (palm-fingers-object system). See Fig. 1. The kinematic chains (limbs) of a parallel manipulator contain active and passive joints. Similarly, in the palm-fingers-object system, a finger in contact with the object represents the kinematic chain where the finger contains active joint(s) and where the contact can be thought as a passive joint.
With: A parallel manipulator does not change its joint positions on the moving platform, but a grasping mechanism can change its positions of contacts on the object. However this difference does not affect the force-closed grasping ability of the grasping mechanism unless some geometric conditions (see Section 3.4) are violated.

Moreover, a parallel manipulator provides immediately a desired dexterity to its moving platform, but the grasping mechanism should first be able to grasp an arbitrary object so that later it can manipulate the object like the parallel manipulator does its moving platform. When the grasped object of

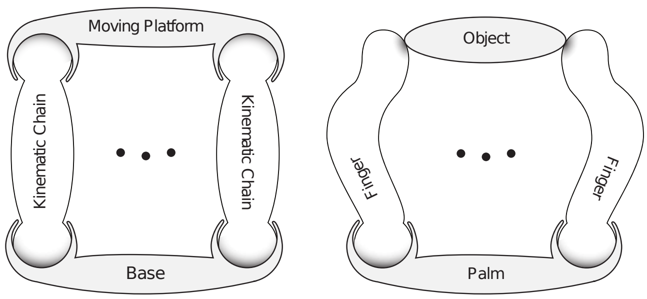


Fig. 1 Topologies of a parallel manipulator and a palm-fingers-object system.

the grasping mechanism is compared with the moving platform of the parallel manipulator from their kinematic pairs point of view, the grasped object is held by the fingertips (*i.e.*, force-closed kinematic pairs) where each fingertip can only push. Whereas the moving platform is held by the joints (*i.e.*, form-closed kinematic pairs) of the limbs of the parallel manipulator where each joint can both push and pull. Consequently, one can conclude that the grasped object can break contact with the fingertips easier than the moving platform can break contact with the limbs of the parallel manipulator against an external disturbing force. In the grasping literature [10], [11], this difference in resistance to external disturbing forces is categorized in two states:

Force-closure grasp: If the fingertips can transmit a desired force and torque to the object by pressing onto it even under certain level of disturbing forces (e.g., gravity, hit by an obstacle), then the grasped object is said to be force-closed. Contacts of a force-closure grasp can resist strong disturbing forces.

Form-closure grasp: Regardless of the applied disturbing forces, if the object cannot be moved without moving any fingertip which presses onto it, then the grasped object is said to be form-closed. In a form-closure grasp, fingertips keep in contact with the object unless you break something.

From above definitions, one can conclude that the grasped object in a dexterous grasping mechanism is rather force-closed, and the moving platform of the parallel manipulator is form-closed.

3 Force-Closure Grasping Ability

The first skill that should be given to a grasping mechanism is the ability to grasp an arbitrary object in a force-closed manner. This skill can be given to the mechanism by satisfying some algebraic and physical conditions on the contacts between the fingertips and the object. Regarding the above statement, the required elements to build a grasping ability are the contact types and their arrangements on the object surface. Here, we proceed first by reintroducing the basic contact types and then by giving the necessary and sufficient conditions for the grasping ability.

3.1 Contacts and Their Mobilities

The relative mobility, α, between two rigid bodies in contact depends on the physics of contact (geometry, friction). Salisbury presented 4 primitive contacts that one can imagine in grasping of an object with a grasping mechanism [3]. He expressed a fingertip-object contact physics as a point-on-plane, a line-on-plane, a plane-on-plane and a soft material-on-plane, with or without friction.

Let a contact frame be located at the center of contact where a fingertip and the object touch each other, and z-axis of the contact frame is aligned with the inward normal vector of the object surface at the center of contact, and the axes x and y are tangent to the object surface. We recall that the relative z-axis translational motion between a fingertip and the object does not exist, since they will be in contact due to the force closure. Now, we can explain the contact types in detail:

Frictionless / frictional point contact: Fingertip and the object touch each other at a single point. The object translates and rotates freely on the fingertip. This can be imagined as a sphere-plane kinematic pair. A point contact thus has 5 independent motions defining the basis:

$$(\mathbb{R}_\alpha) = (\, \mathbf{v}_x,\, \mathbf{v}_y,\, \boldsymbol{\omega}_x,\, \boldsymbol{\omega}_y,\, \boldsymbol{\omega}_z \,)$$

of the vector space of the relative velocities between the fingertip and the object. If enough friction exists, then the tangential translations are restrained completely. Consequently, five motions reduce to 3 rotational motions:

$$(\mathbb{R}_\alpha) = (\, \boldsymbol{\omega}_x,\, \boldsymbol{\omega}_y,\, \boldsymbol{\omega}_z \,)$$

This frictional point contact can then be considered for completely restrained translations as a spherical joint.

Frictionless / frictional line contact: Fingertip and the object touch each other along a finite line segment. The object translates freely on the fingertip and it can only rotate around the line of contact and the normal of contact. A line contact thus has 4 independent motions. If we assume that the x-axis of the contact frame is aligned with the line of contact, then we can write the basis of these 4 independent motions as follows:

$$(\mathbb{R}_\alpha) = (\, \mathbf{v}_x,\, \mathbf{v}_y,\, \boldsymbol{\omega}_x,\, \boldsymbol{\omega}_z \,)$$

This line contact can be considered as either a cylinder-plane or a cylinder-cylinder kinematic pair. If enough friction exists, then the translations and rotations are restrained completely except the rotation around the line of contact. Thus frictional line contact has 1 motion, which is:

$$(\mathbb{R}_\alpha) = (\, \boldsymbol{\omega}_x \,)$$

and it can be considered as a revolute joint.

Frictionless / frictional plane contact: Fingertip and the object touch each other on a planar patch. The object translates freely on the fingertip and rotates around only the normal of contact. Thus plane contact has 3 independent motions:

$$(\mathbb{R}_\alpha) = (\, \mathbf{v}_x,\, \mathbf{v}_y,\, \boldsymbol{\omega}_z \,)$$

This can be imagined as a plane-plane kinematic pair. If enough friction exists, then all the relative motions of the object on the fingertip are restrained completely. Thus frictional plane contact has 0 motion:

$$(\mathbb{R}_\alpha) = \varnothing$$

and it can be considered as a rigid conforming contact.

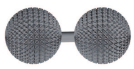
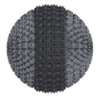
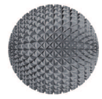
Soft contact: Fingertip is a soft elastic material (*e.g.*, rubber) and touches to the object on a surface patch. The fingertip deforms itself on the object surface such that it forms a special frictional pseudo plane contact where the object can neither translate along the tangential axes of the contact plane nor rotate around the normal of the contact plane. The object is allowed to rotate only around the two tangential axes of the contact plane. Thus soft contact has 2 independent motions:

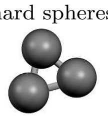
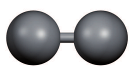
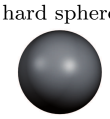
$$(\mathbb{R}_\alpha) = (\boldsymbol{\omega}_x, \boldsymbol{\omega}_y)$$

and it can be considered as a Hooke's joint (universal joint).

We adapt the geometries of the fingertips so that a chosen fingertip can yield a desired contact property as much as possible regardless of the object shape. For example, if the grasped object is a sphere, there is neither a finite line segment nor a planar patch surface to construct line and plane contacts [5]. Amongst the aforementioned contact types, only the contacts with spherical and/or soft geometry (*e.g.*, point contact, soft contact) have strong potential to preserve their contact characteristics with respect to various object shapes. Therefore, we propose to express every fingertip with a set of point or soft contacts. For example, a line contact can be constructed with two (hard, rough/smooth) spheres, and a plane contact with three (soft/hard, rough/smooth) spheres. Table 1 tabulates the contact mobilities ($0 \leqslant \alpha \leqslant 5$) versus their equivalent physics, kinematics and possible fingertip structures.

Table 1 Contact mobilities versus equivalent contact physics, kinematics, and fingertip structures.

Mobility	0	1	2	3
Physics	frictional plane	frictional line	frictional soft	frictional point
Kinematics	rigid	revolute joint	universal joint	spherical joint
Fingertip	three rough & soft spheres 	two rough & hard spheres 	rough & soft sphere 	rough & hard sphere 

Mobility	3	4	5
Physics	frictionless plane	frictionless line	frictionless point
Kinematics	plane-plane pair	cylinder-plane pair	sphere-plane pair
Fingertip	three smooth & hard spheres 	two smooth & hard spheres 	smooth & hard sphere 

3.2 Somov's Necessary Form-Closure Condition for Grasping

Reuleaux showed that it is possible to immobilize an object on the plane with at least 4 frictionless point contacts [12], except that a few special geometric shapes, *e.g.*, circle. Later, Somov showed that using at least 7 frictionless point contacts, we can immobilize an object in 3D space too [13], again except that a few special geometric shapes, *e.g.*, cylinder, sphere. To handle such special geometries frictional contacts should be included into the contact set.

Here, if we assume that each contact occurs between a single fingertip and the object, then we will be able to exploit Somov's result to relate the number of fingers n and contact types to the grasping ability. Now, let $\mathbb{R}_\alpha$ be the relative velocity vector space for the mobility of the object allowed by a fingertip, and let $\mathbb{R}_\beta$ be the transmittable wrench space to the object from a fingertip, where:

$$dim(\mathbb{R}_\alpha) + dim(\mathbb{R}_\beta) = 6 \tag{1}$$

From a kinematics point of view, $\alpha = dim(\mathbb{R}_\alpha)$ indicates the number of independent movements allowed by the fingertip for the mobility of the object (*i.e.*, degrees of mobility). From a statics point of view, $\beta = dim(\mathbb{R}_\beta)$ indicates the number of independent forces transmittable by the fingertip to constrain the object (*i.e.*, degrees of constraint). So, taking into account Somov's form-closure condition, an n-fingered mechanism should impose at least seven constraints on the object:

$$\sum_{i=1}^{n} \beta_i \geqslant 7 \tag{2}$$

where β_i is the degrees of constraint transmitted to the object by the i^{th} fingertip. The equation (2) is just a necessary condition for a grasping ability, but not sufficient. If we assume that the degrees of constraints imposed by each of the fingertips to the object are identical ($\beta_1 = \ldots = \beta_n \equiv \beta$), then equation (2) simplifies to:

$$n\,\beta \geqslant 7 \tag{3}$$

Based on equation (3), we can list the number of fingers versus minimum degrees of constraint by a fingertip that should be imposed to the object as in Table 2.

Table 2 Number of fingers versus minimum degrees of constraint.

Number of Fingers	(n)	2	3	4	5	6	7
Minimum Constraint	(β)	4	3	2	2	2	1

3.3 Sufficient Force-closure Condition for Grasping

The fingertip can only block or push the object. Therefore, to be able to grasp an object, n fingertips' transmittable wrench spaces to the object should span positively the whole wrench space:

$$dim(\mathbb{R}_{\beta_1} \cup \ldots \cup \mathbb{R}_{\beta_n}) = 6\,, \qquad n \geqslant 2 \tag{4}$$

again where $\mathbb{R}_\beta$ is a transmittable wrench space to the object from a fingertip. The above statement means that we can apply any force and torque on the object. So we can grasp it by counterbalancing any wrench acting on it (*e.g.*, gravitational, inertial, external), and more we can also rotate or translate it in any way we want by overbalancing this active wrench. Equation (4) also implies:

$$dim(\mathbb{R}_{\alpha_1} \cap \ldots \cap \mathbb{R}_{\alpha_n}) = 0\,, \qquad n \geqslant 2 \tag{5}$$

again where $\mathbb{R}_\alpha$ is a vector space of the relative velocities of the object allowed by a fingertip. The above statement implies that the object in contact with n fingertips does not have any mutual motion, and therefore the object is immobile. Equation (4) is the sufficient condition for force-closure grasping ability.

3.4 Geometric Conditions for Force-Closure Grasps

From the necessary (2) and sufficient (4) or (5) conditions, and the fingertips shown in Table 1, we can list a few geometric conditions for the relative configurations of the contacts on the object surface. If one of these conditions given below is satisfied, then the mechanism has capacity to grasp an object in a force-closed manner:

1. If there are at least 2 frictional and 2 frictionless point contacts from n fingertips, where $n \geqslant 2$, and such that these 4 point contacts are not planar;
2. If there are at least 3 frictional point contacts from n fingertips, where $n \geqslant 2$, and such that 3 frictional point contacts are not collinear;
3. If there are at least 2 soft contacts from n fingertips, where $n \geqslant 2$, and such that the contact planes of 2 soft contacts are not coplanar; etc.

4 Dexterous Manipulation Ability

The second skill that should be given to a grasping mechanism is that it should be able to impart the desired fine motions to the object through its fingers. The desired fine motions are expressed relatively to the palm of the mechanism. To develop this skill, in the next subsections we outline the dexterity characteristics: mobility, connectivity, redundancy and overconstraint criteria of the palm-fingers-object system by rewriting the new formulas proposed in [8] for parallel manipulators.

4.1 The Palm-Fingers-Object System

Mobility: Assuming that a finger is either an elementary (without closed-loops) or a complex (with closed-loops) kinematic chain; that a finger has a single fingertip, and only the fingertip contacts with the object; that a fingertip and the object always maintain the contact; and that the palm is rigid; then we rewrite the mobility formula proposed in [8] for the palm-fingers-object system as follows:

$$M = \sum_{i=1}^{n} \left(\sum_{k=1}^{m_i} J_k + \alpha_i - r_i \right) - \sum_{i=1}^{n} F_i + X \tag{6}$$

where M is the mobility of the palm-fingers-object system; n is the number of fingers; m is the number of joints in a finger; J is the mobility of a joint; α is the mobility of a contact between a fingertip and the object; r is the number of parameters that loose their independence in the closed loops of the kinematic chain of a finger ($r = 0$ for elementary fingers); F is the connectivity of a finger-plus-contact system; and X is the connectivity of the object to the palm.

Connectivity between the object and the palm: The connectivity between the object and the palm should vary between $1 \leqslant X \leqslant 6$ for the desired fine motions. The connectivity X and the operational velocity vector space $\mathbb{R}_X$ (relative to the palm) of the object are task dependent, and they need to be determined beforehand as a part of the desired dexterity characteristics:

$$X = dim(\mathbb{R}_X), \qquad (\mathbb{R}_X) \subseteq (\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z, \boldsymbol{\omega}_x, \boldsymbol{\omega}_y, \boldsymbol{\omega}_z) \tag{7}$$

Redundancy: Redundancy R of this palm-fingers-object system is given as below:

$$R = M - X - \sum_{i=1}^{n} r_i \tag{8}$$

Overconstraint: Overconstraint O of this palm-fingers-object system is given as below:

$$O = 6\,(n - 1) - \sum_{i=1}^{n} F_i + X - \sum_{i=1}^{n} r_i \tag{9}$$

4.2 A Finger-Plus-Contact System

Mobility: The mobility of a finger-plus-contact system, M_F, is given as follows:

$$M_{F_i} = \sum_{k=1}^{m_i} J_k + \alpha_i - r_i\,, \qquad i = 1, \ldots, n \tag{10}$$

again where m is the number of joints in a finger; J is the mobility of a joint; α is the mobility of a contact; and r is the number of parameters that loose their independence in the closed loops of the kinematic chain of a finger. The mobility of a finger-plus-contact system should satisfy the following condition:

$$M_{F_i} \geqslant M \geqslant X\,, \qquad i = 1, \ldots, n \tag{11}$$

Connectivity: The connectivity F of a finger-plus-contact system should be at least equal to the desired connectivity X of the object to the palm, and that the velocity vector space $\mathbb{R}_F$ of a finger-plus-contact system should at least contain the operational velocity vector space of the object $\mathbb{R}_X$:

$$F_i \geqslant X\,, \quad (\mathbb{R}_{F_i}) \supseteq (\mathbb{R}_X)\,, \quad i = 1, \ldots, n \tag{12}$$

and also the following condition should be satisfied by the velocity vector spaces of finger-plus-contact systems and operational velocity vector space of the object:

$$(\mathbb{R}_X) = (\mathbb{R}_{F_1} \cap \ldots \cap \mathbb{R}_{F_n}) \tag{13}$$

Moreover, between the mobility and connectivity of a finger-plus-contact system the following condition exists:

$$M_{F_i} \geqslant F_i \leqslant 6\,, \quad i = 1, \ldots, n \tag{14}$$

4.3 Redundancy and Overconstraint

Non-redundant, redundant, non-overconstrained, or overconstrained dexterous grasping mechanisms hold respectively the following conditions: *Non-redundant:* $(R = 0)$: If the palm-fingers-object system has the following characteristics:

$$X = M \leqslant M_{F_i}\,, \quad M_{F_i} = F_i \leqslant 6\,, \quad i = 1, \ldots, n \tag{15}$$

then it is non-redundant.

Redundant: $(R > 0)$: If the palm-fingers-object system has the following characteristics:

$$X < M \leqslant M_{F_i}\,, \quad M_{F_i} > F_i \leqslant 6\,, \quad i = 1, \ldots, n \tag{16}$$

then it is redundant.

Non-overconstrained: $(O = 0)$: If mobility M given by (6) is equal to the right-hand side of (17), then the palm-fingers-object system is non-overconstrained:

$$M = \sum_{i=1}^{n} \left(\sum_{k=1}^{m_i} J_k + \alpha_i \right) - 6\,q \tag{17}$$

where q is the number of independent closed loops of the palm-fingers-object system.

Overconstrained: ($O > 0$): If mobility M given by (6) is greater than the right-hand side of (18), then the palm-fingers-object system is overconstrained:

$$M > \sum_{i=1}^{n} \left(\sum_{k=1}^{m_i} J_k + \alpha_i \right) - 6\,q \tag{18}$$

again where q is the number of independent closed loops of the palm-fingers-object system.

5 Evolutionary Morphology

For conceptual design of a palm-fingers-object system, one can use the evolutionary morphology (EM) approach proposed in [8]. EM is formalized with the sets of design objectives, constituent elements, morphological operators, evolution criteria, solutions, and with a termination criteria:

$$EM_t = (\Phi, E, \$, \Psi_t, \Sigma_t, x) \tag{19}$$

where Φ is the set of design objectives such as number of fingers, connectivity of the object, operational velocity vector space of the object, mobility, redundancy, overconstraint of the grasping mechanism, etc.; where E is the set of constituent elements such as joint types (*e.g.*, revolute, universal, prismatic, spherical, parallelogram), contact types, etc.; where \$ is the set of morphological operators, applied to the constituent elements at each generation, such as (re)combination, mutation, migration and selection; where Ψ_t is the set of evolution criteria from generation t to $t+1$ based on the connectivity of the system; where Σ_t is the set of solutions at generation t; and where $x \in \{true, false\}$ is a termination criterion which stops the EM when the set of solutions satisfies the design objectives. EM is a qualitative method and it is oriented for conceptual design. EM investigates complex problems which cannot be treated by direct mathematical formalization. See Algo.1 for the pseudo-code of the EM approach.

The reader is referred to [8] for further details. Using this EM approach, one can define a conceptual design algorithm for a dexterous grasping mechanism as in the next section.

6 Conceptual Design Algorithm

Conceptual design of a dexterous grasping mechanism is closely related to the study of the mechanism's structural parameters used to characterize the parallel manipulators: mobility, connectivity, overconstraint and redundancy. These parameters define the dexterity skill of the mechanism. Here we unify

Algorithm 1 Evolutionary Morphology

define Φ // design criteria
define E // constituent elements
define $\$$ // morphological operators
$x \Leftarrow false$ // termination criterion
$\Sigma = \$.combine(\, E, \varnothing\,)$ // initial generation
while $x \neq true$ **do**
  $\Sigma = \$.combine(\, E, \Sigma\,)$
  $\Psi = evaluate(\, \Sigma\,)$ // evolution criteria
  $\Sigma = \$.mutate(\, \Sigma\,)$
  $\Sigma = \$.migrate(\, \Sigma\,)$
  $\Sigma = \$.select(\, \Sigma, \Psi\,)$
end while
$\Sigma = \$.select(\, \Sigma, \Phi\,)$
return Σ // solution set

mechanism's structural parameters and the geometric conditions of a force-closure grasp, and we propose a conceptual design algorithm where the design of a grasping mechanism evolves from the dexterity skill to the grasping skill:

- **Dexterity skill**:
  1. Choose the desired number of fingers, $n \geq 2$.
  2. Decide the necessary characteristics of fingertips for a grasping ability, see (2) and Table 1.
  3. Choose the desired connectivity, $1 \leqslant X \leqslant 6$, and operational velocity vector space, $\mathbb{R}_X$, of the object for a dextrous manipulation ability, see (7).
  4. If desired, then choose redundancy R and overconstraint O degrees of the palm-fingers-object system. Otherwise, they are chosen as $R \geqslant 0$ and $O \geqslant 0$.
  5. Decide the mobility M_{F_i}, connectivity F_i, and velocity vector space $\mathbb{R}_{F_i}$ of each of the finger-plus-contact systems, where $i = 1, \ldots, n$. See Section 4.3.
  6. Choose the set of joint types that will be used in the finger structures.
  7. Apply EM with the chosen design objectives to obtain each finger-plus-contact system structure such that the fingertip characteristics do not change.
- **Grasping skill**:
  1. Choose the constituent elements as the set of previously obtained n finger-plus-contact systems.
  2. Again apply EM to obtain the dexterous grasping mechanism such that both the desired operational velocity vector space $\mathbb{R}_X$ and one of the geometric conditions for force-closure grasps listed in Section 3.4 are satisfied.

7 Case Studies of Grasping Mechanisms

7.1 A Three-Fingered T3-type Dexterous Grasping Mechanism

We would like to find the concept of a non-redundant grasping mechanism which can perform dexterous grasps with three translational (T3) motions in the Cartesian space.

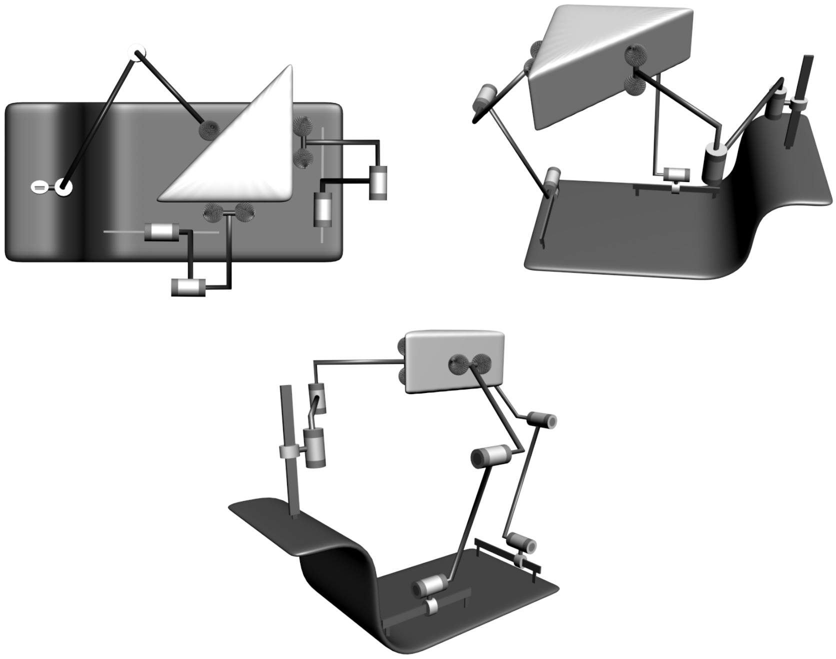


Fig. 2 Dexterous grasp with 3 degrees of connectivity.

Dexterity: (i) We choose to have three, $n = 3$, fingers for this dexterous grasping mechanism. (ii) We decide the characteristics of a fingertip to be a frictional line contact (two rough and hard spheres) which can be imagined as a revolute joint with 1 degree of mobility ($\alpha_i = 1$, where $i = 1, 2, 3$). The degrees of constraint of the three frictional line contacts ($\beta_i = 6 - \alpha_i = 5$) satisfy Somov's (necessary) form-closure condition:

$$\beta_1 + \beta_2 + \beta_3 \geqslant 7 \tag{20}$$

The above statement implies that we can impose enough number of constraints on the object to be able to grasp it. (iii) For the dextrous manipulability of the grasp, we choose the desired connectivity as $X = 3$ with three translational motions:

$$(\mathbb{R}_X) = (\, \mathbf{v}_x,\, \mathbf{v}_y,\, \mathbf{v}_z \,) \tag{21}$$

(iv) We choose the palm-fingers-object system to be non-redundant:

$$R = 0 \tag{22}$$

(v) Here, we will use identical fingers in the conceptual design of the grasping mechanism. Then, we decide the mobility, connectivity and velocity space of each finger-plus-contact system to be, from (12), (15) and (21), as follows:

$$M_{F_i} = F_i\,, \qquad 3 \leqslant F_i \leqslant 6 \tag{23}$$

$$(\mathbb{R}_{F_i}) \supseteq (\mathbb{R}_X) = (\, \mathbf{v}_x,\, \mathbf{v}_y,\, \mathbf{v}_z \,) \tag{24}$$

(vi) In the design of finger structures, we choose to use revolute (R) and prismatic (P) joints. (vii) The evolutionary morphology approach yields a desired finger-plus-contact system as $\{(P) \parallel (R) \parallel (R) \parallel (R)\}$ with one prismatic joint, two revolute joints and one frictional line contact (i.e., revolute (R) joint at the end of the chain), where their motion axes are arranged to be parallel to each other. This corresponds to the following basis of the velocity vector space:

$$(\mathbb{R}_{F_i}) = (\, \mathbf{v}_x,\, \mathbf{v}_y,\, \mathbf{v}_z,\, \boldsymbol{\omega} \,) \tag{25}$$

where the axis of the rotational veloctiy $\boldsymbol{\omega}$ is aligned with the line of contact (or equivalently with the axis of the last revolute joint).

Grasping: (i) We choose the set of constituent elements to be the three identical $\{(P) \parallel (R) \parallel (R) \parallel (R)\}$ structured finger-plus-contact systems. (ii) Then we obtain the grasp configuration using EM such that the solution satisfies both the desired operational velocity space (21) of the object and the second geometric force-closure condition of Section 3.4. The solution is shown in Fig. 2. In the solution, the motion axes of the any two of three prismatic joints, which are attached to the base, are aligned orthogonally to each other.

Remark: We would like to put our finger on the dexterous grasping mechanism shown in Fig. 2. Existing quick mobility formulas (e.g., the well-known Chebychev-Grübler-Kutzbach formula [14], [15], [16]) cannot calculate the correct mobility of this dexterous grasping mechanism, except the formula rewritten in (6).

7.2 A two-fingered planar T2R1-type grasping mechanism

We would like to find the concept of a non-redundant grasping mechanism which can perform dexterous grasps with two translational (T2) and one rotational (R1) motions on the plane.

Dexterity: (i) We choose to have two, $n = 2$, fingers for this dexterous grasping mechanism. (ii) We decide the characteristics of a fingertip to be a frictional line contact (two rough and hard spheres) which can be imagined as a revolute joint with 1 degree of mobility ($\alpha_i = 1$, where $i = 1, 2$). The degrees of constraint of the two frictional line contacts ($\beta_i = 6 - \alpha_i = 5$) satisfy Somov's (necessary) form-closure condition:

$$\beta_1 + \beta_2 \geqslant 7 \tag{26}$$

The above statement implies that we can impose enough number of constraints on the object to be able to grasp it. (iii) For the dexterous manipulability of the grasp, we choose the desired connectivity as $X = 3$ with two translational and one rotational motions:

$$(\mathbb{R}_X) = (\,\mathbf{v}_x,\, \mathbf{v}_y,\, \boldsymbol{\omega}_z\,) \tag{27}$$

where the axis of the rotational velocity $\boldsymbol{\omega}_z$ is orthogonal to the plane of translational velocities $\mathbf{v}_x$ and $\mathbf{v}_y$. (iv) We choose the hand-object system to be non-redundant:

$$R = 0 \tag{28}$$

(v) Here, we will use identical fingers in the conceptual design of the mechanism. Then, we decide the mobility, connectivity and velocity space of each finger-plus-contact system to be, from (12), (15) and (27), as follows:

$$M_{F_i} = F_i = 3\,, \qquad (\mathbb{R}_{F_i}) = (\,\mathbf{v}_x,\, \mathbf{v}_y,\, \boldsymbol{\omega}_z\,) \tag{29}$$

(vi) In the design of finger structures, we choose to use only revolute joints, (R). In this way we ease the concept of the finger-plus-contact system, since the fingertip and the finger will be constructed from same type of joints. (vii) The evolutionary morphology approach yields a desired finger-plus-contact system as $\{(R) \parallel (R) \parallel (R)\}$ with two revolute joints plus one frictional line contact (i.e., revolute (R) joint at the end of the chain), where their axes are arranged to be parallel to each other.

Grasping: (i) We choose the set of constituent elements to be the two $\{(R) \parallel (R) \parallel (R)\}$ structured finger-plus-contact systems. (ii) Then we obtain the grasp configuration using EM such that the solution satisfies both the desired operational velocity vector space (27) of the object and the second geometric force-closure condition of Section 3.4. The solution is shown in Fig. 3.

7.3 A three-fingered T3R3-type grasping mechanism

We would like to find the concept of a non-redundant grasping mechanism which can perform dexterous grasps with three translational (T3) and three rotational (R3) motions in the Cartesian space.

Dexterity: (i) We choose to have three, $n = 3$, fingers for this dexterous grasping mechanism. (ii) We decide the characteristics of a fingertip to be a

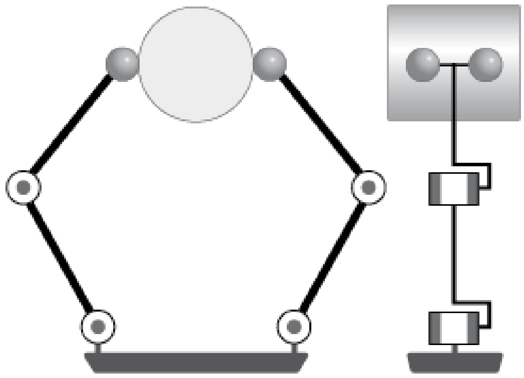


Fig. 3 Planar dexterous grasp with 3 degrees of connectivity.

frictional point contact (a rough and hard sphere) which can be imagined as a spherical joint with 3 degrees of mobility ($\alpha_i = 3$, where $i = 1, 2, 3$). The degrees of constraint of the three frictional point contacts ($\beta_i = 6 - \alpha_i = 3$) satisfy Somov's (necessary) form-closure condition:

$$\beta_1 + \beta_2 + \beta_3 \geqslant 7 \tag{30}$$

The above statement implies that we can impose enough number of constraints on the object to be able to grasp it. (iii) For the dexterous manipulability of the grasp, we choose the desired connectivity as $X = 6$ with three translational and three rotational motions:

$$(\mathbb{R}_X) = (\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z, \boldsymbol{\omega}_x, \boldsymbol{\omega}_y, \boldsymbol{\omega}_z) \tag{31}$$

(iv) We choose the palm-fingers-object system to be non-redundant:

$$R = 0 \tag{32}$$

(v) Here, we will use identical fingers in the conceptual design of the mechanism. Then, we decide the mobility, connectivity and velocity space of each finger-plus-contact system to be, from (12), (15) and (31), as follows:

$$M_{F_i} = F_i = 6, \quad (\mathbb{R}_{F_i}) = (\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z, \boldsymbol{\omega}_x, \boldsymbol{\omega}_y, \boldsymbol{\omega}_z) \tag{33}$$

(vi) In the design of finger structures, we choose to use only revolute joints, (R). (vii) The evolutionary morphology approach yields a desired finger-plus-contact system as a parallelogram plus the frictional point contact fingertip structure (i.e., a spherical (S) joint at the end of the chain) ($\{(R) \perp (R) \perp^{\parallel} (R)\} \parallel \{(R) \perp (R) \perp^{\parallel} (R)\}) - (S)$.

Grasping: (i) We choose the set of constituent elements to be the three identical ($\{(R) \perp (R) \perp^{\parallel} (R)\} \parallel \{(R) \perp (R) \perp^{\parallel} (R)\}) - (S)$ structured finger-plus-contact systems. (ii) Then we obtain the grasp configuration using EM such that the solution satisfies both the desired operational velocity vector space

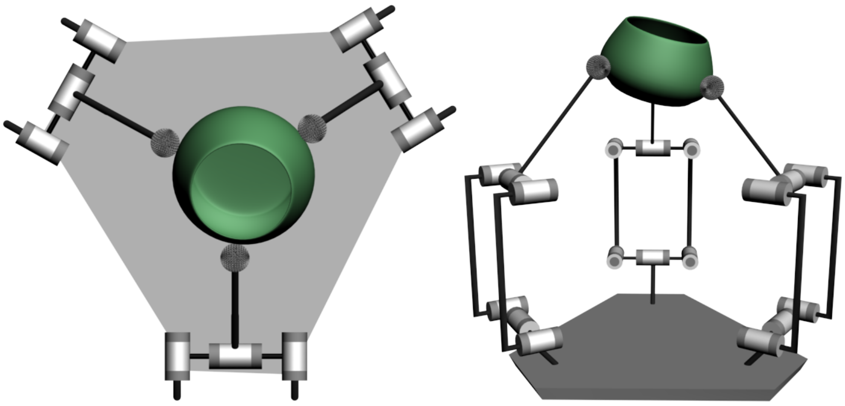


Fig. 4 Dexterous grasp with 6 degrees of connectivity.

(31) of the object and the second geometric force-closure condition of Section 3.4. The solution is shown in Fig. 4.

With respect to the previous three-fingered grasping mechanisms illustrated in this section, the three-fingered T3R3-type grasping mechanism despite its higher structural complexity presents the following advantages: (i) larger dexterous working space for similar dimensions of the links, (ii) higher finger rigidity due to the parallelogram mechanism and (iii) higher grasping ability due to the larger mobility of the finger-plus-contact system.

8 Conclusion

In this paper we studied the conceptual design principles of dexterous grasping mechanisms from the parallel manipulators framework. We also proposed an innovative conceptual design algorithm with the desired dexterity characteristics: mobility, connectivity, overconstraint and redundancy. To the best of our knowledge, this is the first algorithm which takes into account all the dexterity characteristics in the conceptual design of a dexterous grasping mechanism. The future work of this paper will concentrate on the optimization of dimensions and dexterous workspace of such grasping mechanisms and then their construction. The main sources of error such as link dimensional error, position of joint axes error, geometric and friction contact type error, force-closure grasping error will be considered in the next investigations.

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