Structural Synthesis of Dexterous Hands

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***Abstract*— This paper proposes a complete procedure for the structural synthesis of dexterous hands. This procedure fuses the theories already developed for the structural synthesis of dexterous hand and parallel robots. Unlike others, this procedure allows one to synthesize any kind of dexterous hand with the desired structural design parameters: mobility, connectivity, overconstraint, and redundancy. Two examples of dexterous hands, which are synthesized to have 3 dof planar motion and 6 dof spatial motion, are also given.**

I. INTRODUCTION

Dexterous hands are rarely studied in the literature from structural synthesis point of view, and yet to be given a complete method for their synthesis. Whereas hand-plus-object closed-loop system has huge structural similarities to the well studied parallel robots. In this paper thus we build upon the seminal works done for the structural synthesis of dexterous hands by borrowing the ideas of parallel robotics.

The seminal works on the structural synthesis of dexterous hands can be found in [1], [2], [3], [4], [5]. Salisbury in [1], [2] used Grübler's mobility formula and synthesized a three-fingered hand with frictional point contacts yielding 6 degrees-of-freedom. Later, Tischler et. al. [3] proposed a new Melbourne method to synthesize complete list of kinematic chains for robot hands. They showed that their method produces the least number of isomorphic chains in the complete list compared to other existing methods. This method was based on Freudenstein's mobility formula [6] and screw system theory. Tischler et. al. [4] showed how to use this Melbourne method to synthesize fingers of a robot hand by taking into account the kinematic constraints imposed by the grasping and manipulation of an arbitrary object. Fingers, that they synthesized, (i) consider only frictional point contacts; (ii) contain one dof joints. Then, Lee and Tsai in [5] used Tsai's mobility formula to synthesize n-fingered hands ($3 \leqslant n \leqslant 7$) with different types of contact yielding an atlas of feasible multi-fingered hand structures with mobilities three to six. Fingers, that they synthesized, also contain only one dof joints.

The main drawback of these aforementioned methods that their mobility calculation formulas are not valid for all kind of mechanisms, and thus the proposed set of feasible hand structures are incomplete and may contain erroneous solutions. Recently in [7], Gogu reformulated properly the quick mobility calculation formula, which is valid for all mechanisms. Gogu also gave formulas for the connectivity, overconstraint, and redundancy of a mechanism. Here we adopt these formulas. Thus, this paper contributes for the structural synthesis of dexterous hands in two ways: (i) it proposes a complete procedure which takes into account all the structural design parameters: mobility, connectivity, overconstraint, and redundancy; and (ii) it allows to fix the bugs of other dexterous hand synthesis methods with the correct mobility formula of mechanisms.

The rest of this paper goes on as follows: Section II compares robotic hands to and with parallel robots; Sections III and IV explain force-closed grasping and dexterous manipulation abilities of a robotic hand, respectively; Section V presents briefly the evolutionary morphology method (EM); Section VI gives the procedure for structural synthesis of dexterous hands which is based on previously presented EM; Section VII shows the two examples of dexterous hands synthesized from the proposed procedure; and finally Section VIII concludes the paper.

II. COMPARED TO AND WITH PARALLEL ROBOTS

To: A parallel robot has a closed-loop topology, so does a robot hand with the grasped object (palm-fingers-object system). See Fig. 1. The kinematic chains (limbs) of a parallel robot contain active and passive joints. Similarly, in the palm-fingers-object system, a finger in contact with the object represents the kinematic chain where the finger contains active joint(s) and where the contact can be thought as a passive joint.

With: A parallel robot's main functionality is to provide a desired mobility to its moving platform. When a robot hand is compared with a parallel robot from functionality point of view, the robot hand should first be able to grasp an arbitrary object so that later it can manipulate the object like the parallel robot does its moving platform. When the grasped object of the robot hand is compared with the moving platform of the parallel robot from their kinematic pairs point of view, the grasped object is held by the fingertips (*i.e.*, force-closed kinematic pairs) where each fingertip can only push. Whereas the moving platform is held by the joints (*i.e.*, form-closed kinematic pairs) of the limbs of the parallel

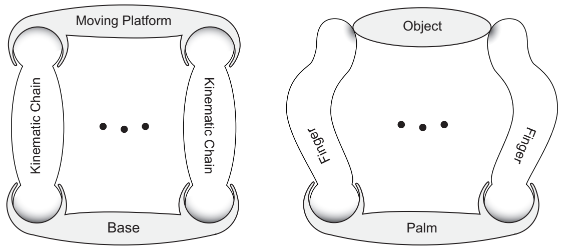

Fig. 1. Topologies of a parallel robot and a palm-fingers-object system.

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robot where each joint can both push and pull. Consequently, one can conclude that the grasped object can break contact with the fingertips easier than the moving platform can break contact with the limbs of the parallel robot against an external disturbing force. In the grasping literature [8], [9], this difference in resistance to external disturbing forces is categorized in two states:

Force-closure grasp: If the fingertips can transmit a desired force and torque to the object by pressing onto it even under some certain level of disturbing forces (e.g., gravity, hit by an obstacle), then the grasped object is said to be force-closed. Contacts of a force-closure grasp can resist only moderate level of disturbing forces.

Form-closure grasp: Regardless of the applied disturbing forces, if the object cannot be moved without moving any fingertip which presses onto it, then the grasped object is said to be form-closed. Contacts of a form-closure grasp can resist strong disturbing forces, unless you break something.

From above definitions, one can conclude that the grasped object in a dexterous hand is rather force-closed, and the moving platform of the parallel robot is form-closed.

III. Force-Closure Grasping Ability

The first skill that should be given to a robot hand is the ability to grasp an arbitrary object in a force-closed manner. This skill can be given to the hand by satisfying some algebraic and physical conditions on the contacts between the fingertips and the object. Regarding the above statement, the structural elements to build a grasping ability are the contact types and their arrangements on the object surface. We proceed here first by reintroducing the basic contact types and then by giving the necessary and sufficient conditions for grasping ability.

A. Contacts and Their Mobilities

The relative mobility, α, between two rigid bodies in contact depends on the physics of contact (geometry, friction). Salisbury presented 4 primitive contacts that one can imagine in grasping of an object with a mechanical hand [2]. He expressed a fingertip-object contact physics as a point-on-plane, a line-on-plane, a plane-on-plane and a soft material-on-plane, with or without friction.

Let a contact frame be located at the center of contact where a fingertip and the object touch each other, and z-axis of the contact frame is aligned with the inward normal vector of the object surface at the center of contact, and the axes x and y are tangent to the object surface. We recall that the relative z-axis motion between a fingertip and the object does not exist, since they will be in contact due to force closure. Now, we can explain the contact types in detail:

Frictionless/frictional point contact: Fingertip and the object touch each other at a single point. The object translates and rotates freely on the fingertip. This can be imagined as a sphere-plane kinematic pair. Thus point contact has 5 motions: $\mathbb{R}_\alpha = (\mathbf{v}_x, \mathbf{v}_y, \omega_x, \omega_y, \omega_z)$. If enough friction exists, then the tangential translations are either restrained completely or exist infinitesimally. Consequently, five motions reduce to 3 rotational motions: $\mathbb{R}_\alpha = (\omega_x, \omega_y, \omega_z)$. This frictional point contact can then be considered for completely restrained translations as an ideal spherical joint or for existing infinitesimal translations as a spherical joint with some backlashes.

Frictionless/frictional line contact: Fingertip and the object touch each other along a finite line segment. The object translates freely on the fingertip and it can only rotate around the line of contact and the normal of contact. Thus line contact has 4 motions. If we assume that the x-axis of the contact frame is aligned with the line of contact, then we can write these 4 motions as follows: $\mathbb{R}_\alpha = (\mathbf{v}_x, \mathbf{v}_y, \omega_x, \omega_z)$. This line contact can be considered as either a cylinder-plane or a cylinder-cylinder kinematic pair. If enough friction exists, then the translations and rotations are either restrained completely or exist infinitesimally, except the rotation around the line of contact. Thus frictional line contact has 1 motion, which is $\mathbb{R}_\alpha = (\omega_x)$, and it can be considered as a revolute joint.

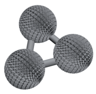
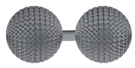
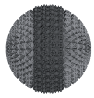
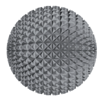
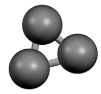
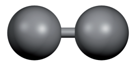
Frictionless/frictional plane contact: Fingertip and the object touch each other on a planar patch. The object translates freely on the fingertip and rotates around only the normal of contact. Thus plane contact has 3 motions: $\mathbb{R}_\alpha = (\mathbf{v}_x, \mathbf{v}_y, \omega_z)$. This can be imagined as a plane-plane kinematic pair. If enough friction exists, then all the relative motions of the object on the fingertip are either restrained completely or exist infinitesimally. Thus frictional plane contact has 0 motion: $\mathbb{R}_\alpha = \varnothing$, and it can be considered as a rigid conforming contact.

Soft contact: Fingertip is a soft elastic material (*e.g.*, rubber) and touches to the object on a surface patch. The fingertip deforms itself on the object surface such that it forms a special frictional pseudo plane contact where the object can neither translate along the tangential axes of the contact plane nor rotate around the normal of the contact plane. The object is allowed to rotate only around the two tangential axes of the contact plane. Thus soft contact has 2 motions: $\mathbb{R}_\alpha = (\omega_x, \omega_y)$, and it can be considered as a Hooke's joint (universal joint).

We adapt the geometries of the fingertips so that a chosen fingertip can yield a desired contact property as much as possible regardless of the object shape. For example, if the grasped object is a sphere, there is neither a finite line segment nor a planar patch surface to construct line and plane contacts [4]. Amongst the aforementioned contact types, only the contacts with spherical and/or soft geometry (*e.g.*, point contact, soft contact) have strong potential to preserve their contact characteristics with respect to various object shapes. Therefore, we propose to express every fingertip with a set of point or soft contacts. For example, a line contact can be constructed with two (hard, rough/smooth) spheres, and a plane contact with three (soft/hard, rough/smooth) spheres. Table I tabulates the contact mobilities ($0 \leqslant \alpha \leqslant 5$) versus their equivalent physics, kinematics and possible fingertip structures.

TABLE I

CONTACT MOBILITIES VERSUS EQUIVALENT CONTACT PHYSICS, KINEMATICS, AND FINGERTIP STRUCTURES.

Mobility	0	1	2	3	3	4	5
Physics	frictional plane	frictional line	frictional soft	frictional point	frictionless plane	frictionless line	frictionless point
Kinematics	rigid	revolute joint	universal joint	spherical joint	plane-plane pair	cylinder-plane pair	sphere-plane pair
Fingertip	three rough & soft spheres 	two rough & hard spheres 	rough & soft sphere 	rough & hard sphere 	three smooth & hard spheres 	two smooth & hard spheres 	smooth & hard sphere 

B. Somov's Necessary Form-Closure Condition for Grasping

Reuleaux showed that it is possible to immobilize an object on the plane with at least 4 frictionless point contacts [10], except that a few special geometric shapes, *e.g.*, circle. Later, Somov showed that using at least 7 frictionless point contacts, we can immobilize an object in 3D space too [11], again except that a few special geometric shapes, *e.g.*, cylinder, sphere. Here, if we assume that each contact type is performed by a single finger, then we will be able to exploit Somov's result to relate the number of fingers n and contact types to the grasping ability. Now, let α and β be respectively the degrees of mobility and the degrees of constraint of a contact formed between the object and a fingertip, where:

$$\alpha + \beta = 6 \tag{1}$$

From a kinematics point of view, α indicates the number of independent movements allowed by the fingertip for the mobility of the object. From a statics point of view, β indicates the number of independent forces transmittable by the fingertip to constrain the object. So, taking into account Somov's form-closure condition, an n-fingered hand should impose at least seven constraints on the object:

$$\sum_{i=1}^{n} \beta_i \geqslant 7 \tag{2}$$

where β_i is the degrees of constraint transmitted to the object by the i^{th} fingertip. The equation (2) is just a necessary condition for a grasping ability, but not sufficient. If we assume that the degrees of constraints imposed by each of the fingertips to the object are identical ($\beta_1 = \ldots = \beta_n \equiv \beta$), then equation (2) simplifies to:

$$n\beta \geqslant 7 \tag{3}$$

Based on equation (3), we can list the number of fingers versus minimum degrees of constraint by a fingertip that should be imposed to the object as in Table II.

TABLE II

NUMBER OF FINGERS VERSUS MINIMUM DEGREES OF CONSTRAINT.

Number of Fingers (n)	2	3	4	5	6	7
Minimum Constraint (β)	4	3	2	2	2	1

C. Sufficient Force-closure Condition for Grasping

The fingertip can only block or push the object. Therefore, to be able to grasp an object, n fingertips' transmittable wrench spaces to the object should span positively the whole wrench space:

$$dim(\mathbb{R}_{\beta_1} \cup \ldots \cup \mathbb{R}_{\beta_n}) = 6, \qquad n \geqslant 2 \tag{4}$$

where $\mathbb{R}_\beta$ is a transmittable wrench space to the object from a fingertip. The above statement means that we can apply any force and torque on the object. So we can grasp it by counterbalancing any wrench acting on it (*e.g.*, gravity, inertia, external), and more we can also rotate or translate it in any way we want by overbalancing this active wrench. Equation (4) also implies:

$$dim(\mathbb{R}_{\alpha_1} \cap \ldots \cap \mathbb{R}_{\alpha_n}) = 0, \qquad n \geqslant 2 \tag{5}$$

where $\mathbb{R}_\alpha$ is an allowed velocity space for the mobility of the object by a fingertip. The above statement says that the allowed mobilities of the object from n fingertips do not have any mutual motion in their velocity spaces, and therefore the object is immobile. Equation (4) is the sufficient condition for force-closure grasping ability.

D. Geometric Conditions for Force-Closure Grasps

From the necessary (2) and sufficient (4), (5) conditions, and the fingertips shown in Table I, we can list a few geometric conditions for the relative configurations of the contacts on the object surface. If one of these conditions given below is satisfied, then the robot hand has capacity to grasp an object in a force-closed manner:

1) If there are at least 2 frictional and 2 frictionless point contacts from n fingertips, where $n \geqslant 2$, and such that these 4 point contacts are not planar;
2) If there are at least 3 frictional point contacts from n fingertips, where $n \geqslant 2$, and such that 3 frictional point contacts are not collinear;
3) If there are at least 2 soft contacts from n fingertips, where $n \geqslant 2$, and such that the contact planes of 2 soft contacts are not coplanar.

IV. DEXTEROUS MANIPULATION ABILITY

The second skill that should be given to a robot hand is that it should be able to impart the desired fine motions to the object through its fingers. The desired fine motions are expressed relatively to the palm. To develop this skill, in the next subsections we outline the structural parameters: mobility, connectivity, redundancy and overconstraint criteria of the palm-fingers-object system by rewriting the new formulas proposed in [7] for parallel robots.

A. The Palm-Fingers-Object System

Mobility: Assuming that a finger is either an elementary (without loops) or a complex (with loops) kinematic chain; that a finger has a single fingertip, and only the fingertip contacts with the object; that a fingertip and the object always maintain the contact; and that the palm is rigid; then we rewrite the mobility formula proposed in [7] for the palm-fingers-object system as follows:

$$M = \sum_{i=1}^{n} \left(\sum_{k=1}^{m_i} J_k + \alpha_i - r_i \right) - \sum_{i=1}^{n} F_i + X \qquad (6)$$

where M is the mobility of the palm-fingers-object system; n is the number of fingers; m is the number of joints in a finger; J is the mobility of a joint; α is the mobility of a contact between a fingertip and the object; r is the number of parameters that loose their independence in the closed loops of the kinematic chain of a finger ($r = 0$ for elementary fingers); F is the connectivity of a finger-plus-contact system; and X is the connectivity of the object to the palm.

Connectivity between the Object and the Palm: The connectivity between the object and the palm should vary between $1 \leqslant X \leqslant 6$ for the desired fine motions. The connectivity X and the operational velocity space $\mathbb{R}_X$ (relative to the palm) of the object are task dependent, and they need to be determined beforehand for the structural synthesis:

$$X = dim(\mathbb{R}_X), \qquad \mathbb{R}_X \subseteq (\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z, \omega_x, \omega_y, \omega_z) \qquad (7)$$

Redundancy: Redundancy R of this palm-fingers-object system are given as below:

$$R = M - X - \sum_{i=1}^{n} r_i \qquad (8)$$

Overconstraint: Overconstraint O of this palm-fingers-object system are given as below:

$$O = 6(n-1) - \sum_{i=1}^{n} F_i + X - \sum_{i=1}^{n} r_i \qquad (9)$$

B. A Finger-Plus-Contact System

Mobility: The mobility of a finger-plus-contact system, M_F, is given as follows:

$$M_{F_i} = \sum_{k=1}^{m_i} J_k + \alpha_i - r_i, \qquad i = 1, \ldots, n \qquad (10)$$

again where m is the number of joints in a finger; J is the mobility of a joint; α is the mobility of a contact; and r is the number of parameters that loose their independence in the closed loops of the kinematic chain of a finger. The mobility of a finger-plus-contact system should satisfy the following condition:

$$M_{F_i} \geqslant M \geqslant X, \qquad i = 1, \ldots, n \qquad (11)$$

Connectivity: The connectivity F of a finger-plus-contact system should be at least equal to the desired connectivity X of the object to the palm, and that the velocity space $\mathbb{R}_F$ of a finger-plus-contact system should at least contain the operational velocity space of the object $\mathbb{R}_X$:

$$F_i \geqslant X, \quad \mathbb{R}_{F_i} \supseteq \mathbb{R}_X, \quad i = 1, \ldots, n \qquad (12)$$

and also the following condition should be satisfied by the velocity spaces of finger-plus-contact systems and operational velocity space of the object:

$$\mathbb{R}_X = (\mathbb{R}_{F_1} \cap \ldots \cap \mathbb{R}_{F_n}) \qquad (13)$$

Moreover, between the mobility and connectivity of a finger-plus-contact system the following condition exists:

$$M_{F_i} \geqslant F_i \leqslant 6, \quad i = 1, \ldots, n \qquad (14)$$

C. Redundancy and Overconstraint

To synthesize non-redundant or redundant, and non-overconstrained or overconstrained dexterous hands the following conditions should be satisfied:

Non-redundant: ($R = 0$): If the palm-fingers-object system has the following characteristics:

$$X = M \leqslant M_{F_i}, \quad M_{F_i} = F_i \leqslant 6, \quad i = 1, \ldots, n \qquad (15)$$

then it is non-redundant.

Redundant: ($R > 0$): If the palm-fingers-object system has the following characteristics:

$$X < M \leqslant M_{F_i}, \quad M_{F_i} > F_i \leqslant 6, \quad i = 1, \ldots, n \qquad (16)$$

then it is redundant.

Non-overconstrained: ($O = 0$): If mobility M given by (6) is equal to the right-hand side of (17), then the palm-fingers-object system is non-overconstrained:

$$M = \sum_{i=1}^{n} \left(\sum_{k=1}^{m_i} J_k + \alpha_i \right) - 6q \qquad (17)$$

where q is the number of independent closed loops of the palm-fingers-object system.

Overconstrained: ($O > 0$): If mobility M given by (6) is greater than the right-hand side of (18), then the palm-fingers-object system is overconstrained:

$$M > \sum_{i=1}^{n} \left(\sum_{k=1}^{m_i} J_k + \alpha_i \right) - 6q \qquad (18)$$

again where q is the number of independent closed loops of the palm-fingers-object system.

V. Evolutionary Morphology

For the structural synthesis of a palm-fingers-object system, one can use the evolutionary morphology (EM) approach proposed in [7]. EM is formalized with the sets of design objectives, constituent elements, morphological operators, evolution criteria, solutions, and with a termination criteria:

$$EM_t = (\Phi, E, \$_t, \Psi_t, \Sigma_t, x) \qquad (19)$$

where Φ is the set of design objectives such as number of fingers, connectivity of the object, operational velocity of the object, mobility, redundancy, overconstraint of the hand-object system, etc.; where E is the set of constituent elements such as joint types (*e.g.*, revolute, universal, prismatic, spherical, parallelogram), contact types, etc.; where $\$_t$ is the set of morphological operators, applied to the constituent elements at each generation t, such as (re)combination, mutation, migration and selection; where Ψ_t is the set of evolution criteria from generation t to $t+1$ based on the connectivity of the system; where Σ_t is the set of solutions at each generation t; and where $x \in \{true, false\}$ is a termination criterion which stops the EM when the set of solutions satisfies the design objectives. EM is a qualitative method and it is oriented for conceptual design. EM investigates complex problems which cannot be treated by direct mathematical formalization. The reader is referred to [7] for further details. Using this EM approach, one can define a procedure for structural synthesis of a dexterous hand as in the next section.

VI. Structural Synthesis Evolving from Dexterity to Grasping

In this procedure, the structural synthesis of a robot hand evolves from dexterity to grasping:

- **Dexterity**:
  1) Choose the desired number of fingers, $n \geq 2$.
  2) Decide the necessary characteristics of fingertips for a grasping ability, see (2) and Table I.
  3) Choose the desired connectivity, $1 \leqslant X \leqslant 6$, and operational velocity space, $\mathbb{R}_X$, of the object for a dextrous manipulation ability, see (7).
  4) If desired, then choose redundancy R and overconstraint O degree of the hand-object system.
  5) Decide the mobility M_{F_i}, connectivity F_i, and velocity space $\mathbb{R}_{F_i}$ of each of the finger-plus-contact systems, where $i = 1, \ldots, n$. See Section IV-C.
  6) Choose the set of joint types that will be used in the finger structures.
  7) Synthesize each finger-plus-contact structure using EM with the chosen design objectives such that the fingertip characteristics do not change.
- **Grasping**:
  1) Choose the constituent elements as the set of synthesized n finger-plus-contact systems.
  2) Synthesize the dexterous hand using EM such that both the desired operational velocity space $\mathbb{R}_X$ and one of the geometric conditions for force-closure grasps listed in Section III-D are satisfied.

VII. Examples of Dexterous Hands

A. A two-fingered planar T2R1-type dexterous hand

We would like to synthesize a non-redundant dexterous hand which can perform manipulable grasps with two translational (T2) and one rotational (R1) motions on the plane.

Dexterity: (i) We choose to have two, $n = 2$, fingers for this dexterous hand. (ii) We decide the characteristics of a fingertip to be a frictional line contact (two rough and hard spheres) which can be imagined as a revolute joint with 1 degree of mobility ($\alpha_i = 1$, where $i = 1, 2$). The degrees of constraint of the two frictional line contacts ($\beta_i = 6 - \alpha_i = 5$) satisfy Somov's (necessary) form-closure condition:

$$\beta_1 + \beta_2 \geqslant 7 \tag{20}$$

The above statement implies that we can impose enough number of constraints on the object to be able to grasp it. (iii) For the dextrous manipulability of the grasp, we choose the desired connectivity as $X = 3$ with two translational and one rotational motions:

$$\mathbb{R}_X = (\mathbf{v}_x, \mathbf{v}_y, \omega_z) \tag{21}$$

where the axis of the rotational velocity ω_z is orthogonal to the plane of translational velocities $\mathbf{v}_x$ and $\mathbf{v}_y$. (iv) We choose the hand-object system to be non-redundant, $R = 0$. (v) Here, we will use identical fingers for the synthesis of the hand. Then, we decide the mobility, connectivity and velocity space of each finger-plus-contact system to be, from (12), (15) and (21), as follows:

$$M_{F_i} = F_i = 3, \quad \mathbb{R}_{F_i} = (\mathbf{v}_x, \mathbf{v}_y, \omega_z) \tag{22}$$

(vi) In the synthesis of finger structures, we choose to use only revolute joints, (R). In this way we ease the synthesis of the finger-plus-contact system, since the fingertip and the finger will be constructed from same type of joints. (vii) The evolutionary morphology approach yields a desired finger-plus-contact system as $\{(R) \parallel (R) \parallel (R)\}$ with two revolute joints plus one frictional line contact where their axes are arranged to be parallel to each other.

Grasping: (i) We choose the set of constituent elements to be the two $\{(R) \parallel (R) \parallel (R)\}$ structured finger-plus-contact systems. (ii) Then we synthesize the grasp configuration using EM such that the solution satisfies both the desired operational velocity space (21) of the object and the second structural force-closure condition of Section III-D. The solution is shown in Fig. 2.

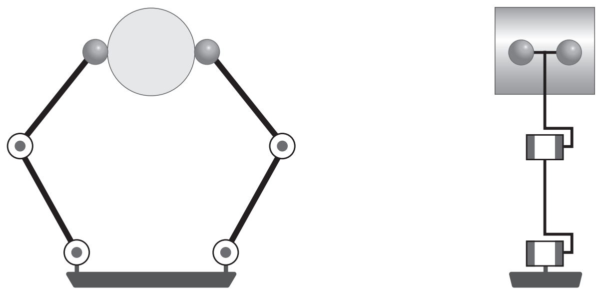


Fig. 2. Planar manipulable grasp with 3 degrees of connectivity.

B. A three-fingered T3R3-type dexterous hand

We would like to synthesize a non-redundant dexterous hand which can perform manipulable grasps with three translational (T3) and three rotational (R3) motions in the Cartesian space.

Dexterity: (i) We choose to have three, $n = 3$, fingers for this dexterous hand. (ii) We decide the characteristics of a fingertip to be a frictional point contact (a rough and hard sphere) which can be imagined as a spherical joint with 3 degrees of mobility ($\alpha_i = 3$, where $i = 1, 2, 3$). The degrees of constraint of the three frictional point contacts ($\beta_i = 6 - \alpha_i = 3$) satisfy Somov's (necessary) form-closure condition:

$$\beta_1 + \beta_2 + \beta_3 \geqslant 7 \tag{23}$$

The above statement implies that we can impose enough number of constraints on the object to be able to grasp it. (iii) For the dextrous manipulability of the grasp, we choose the desired connectivity as $X = 6$ with three translational and three rotational motions:

$$\mathbb{R}_X = (\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z, \omega_x, \omega_y, \omega_z) \tag{24}$$

(iv) We choose the hand-object system to be non-redundant, $R = 0$. (v) Here, we will use identical fingers for the synthesis of the hand. Then, we decide the mobility, connectivity and velocity space of each finger-plus-contact system to be, from (12), (15) and (24), as follows:

$$M_{F_i} = F_i = 6, \qquad \mathbb{R}_{F_i} = (\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z, \omega_x, \omega_y, \omega_z) \tag{25}$$

(vi) In the synthesis of finger structures, we choose to use only revolute joints, (R). (vii) The evolutionary morphology approach yields a desired finger-plus-contact system as a parallelogram plus the frictional point contact fingertip structure $(\{(R) \perp (R) \perp^{\parallel} (R)\} \parallel \{(R) \perp (R) \perp^{\parallel} (R)\}) - (S)$.

Grasping: (i) We choose the set of constituent elements to be the three identical $(\{(R) \perp (R) \perp^{\parallel} (R)\} \parallel \{(R) \perp (R) \perp^{\parallel} (R)\}) - (S)$ structured finger-plus-contact systems. (ii) Then we synthesize the grasp configuration using EM such that the solution satisfies both the desired operational velocity space (24) of the object and the second structural force-closure condition of Section III-D. The solution is shown in Fig. 3.

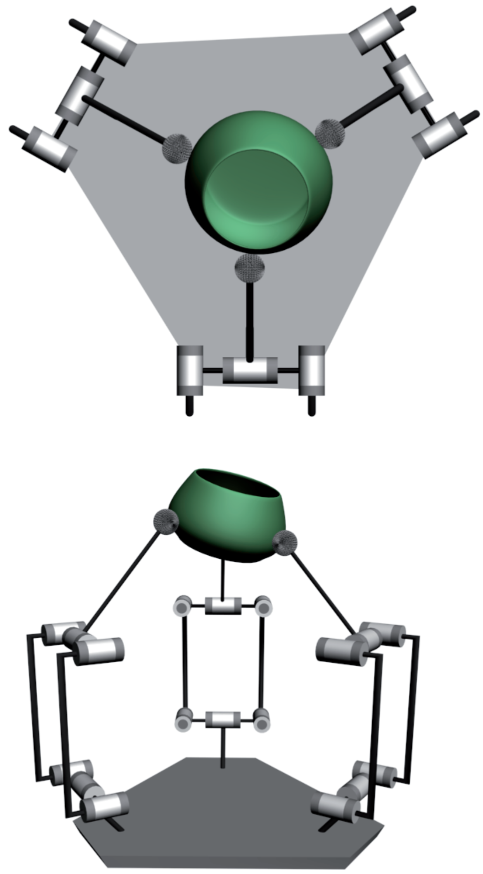


Fig. 3. Manipulable grasp with 6 degrees of connectivity.

VIII. CONCLUSIONS

The presented structural synthesis procedure can produce all possible dextrous hands with the desired number of fingers and with the desired structural dexterity parameters: mobility, connectivity, overconstraint and redundancy. To the best of our knowledge, this is the first procedure which takes into account all the structural design parameters for synthesis of a dexterous hand.

APPENDIX - TERMINOLOGY [7]

Mobility: The number of independent coordinates required to define the configuration of a mechanism (*e.g.*, palm-fingers-object system). It is also called *degrees of freedom.*

Connectivity: The number of independent displacements (finite, infinitesimal) allowed between the two links of a mechanism (*e.g.*, fingers between the palm and the object).

Overconstraint: The difference between the total number of coordinates that could loose their independence before forming a closed-loop mechanism and the number of dependent coordinates after forming a closed-loop mechanism.

Redundancy: The difference between the mobility of the mechanism and the connectivity of its moving platform (*resp.*, the grasped object).

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