Control-Based Synthesis and Tracking of Grasping Points

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***Abstract*— We synthesize grasping points for planar grasps. We formulate and solve the problem from control theory point of view. Designed control law moves initial arbitrary contact points smoothly from a non force-closure configuration to a closest force-closure one. Control law is independent from the friction coefficient, and it is robust to small changes in shape and pose of the object, and it can track a force closure configuration on the object when the object moves and deforms. Final control error yields a quantitative measure for the solution. We also impose unilateral constraints to the control law to eliminate solutions which are not feasible for the hand. We finally show how to synchronize synthesis of grasping points and the reaching motion of the arm-hand manipulator for grasping. This melts the computation time of the synthesis of grasping points in the reaching motion of the manipulator. We validated the proposed approach on real images of the objects.**

I. INTRODUCTION

In this paper, we reformulate the synthesis of grasping points for two-fingered frictional force-closure planar grasps. Synthesis of grasping points is to decide where to place fingertips on an object so that we can hold it firmly. This is an inverse problem, and it has usually multiple solutions depending on the object geometry. Thus finding an optimal solution for grasping points is still one of the important problems of object grasping. One can read more on grasping in [1], [2], [3], [4].

Additionally, a grasp is supposed to be planar if grasping points of the grasp and their normal vectors stay on a plane π. As long as there exists a plane such π on the object, the object can be grasped planarly. This means that the object can be either relatively thinner in one dimension (but still thick enough to be grasped) or large in all dimensions. Fig. 1 shows a sphere and a vase which contain π-like planes. For example, a sphere can be grasped planarly from any of its great circles where the sphere is intersected by a plane passing through its center. The vase shown in Fig. 1.b contains two horizontal planar grasping planes — solid lines.

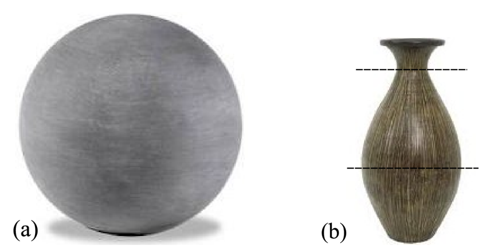

Fig. 1. A sphere can be grasped planarly from any of its great circles (a). A vase and its possible planar grasping planes — solid lines (b).

Here, we synthesize grasping points from a control point of view, and this has strong advantages.

Firstly, it has a coherent framework with control theory of a robotic manipulator. We can exploit all the control theory of robotics for our use. During synthesis of force-closure grasping points, we choose arbitrarily located two non-force closure contact points, and then move them like two mobile robots on the curved road of the object boundary. Designed control law drives the contact points smoothly and parks them to the closest force-closure configuration. This approach brings two more advantages: (i) In every grasp planning once the force-closure grasping points are found on the object, the next step for an arm-hand manipulator is to reach from an initial posture to these desired grasping points. Here, we have chance to synchronize both the synthesis of grasping points and the reaching motion of the manipulator. Imagine that the smooth trajectories of the contact points during synthesis are given as reference inputs to the arm-hand manipulator to guide it from its initial posture to the desired final grasping points on the object. In other words, this means that while the contact points are traveling to a force-closure configuration, arm-hand manipulator is just pursuing behind with a proper tracking latency. Once the contact points are parked to a final force-closure configuration, the fingers of the arm-hand manipulator are straightaway there to grasp the object. Thus, the computational time of the synthesis of the grasping points is melted completely in the reaching motion of the manipulator. Otherwise, a control iteration, which moves contact points to a force-closure configuration, takes $1/2500$ seconds [1]. Thus a force-closure solution with 100 iterations will take $1/25$ seconds. (ii) Imagine that during synthesis of grasping points the object deforms and changes its Euclidean pose continuously. The stable control law still drives the contact points to a force closure configuration. Once the contact points are on a force closure configuration, the control law keeps them there even though the object is still deforming and moving in the scene. Thus proposed control-based approach is robust to disturbances and can track the force closure configuration on the moving object.

Secondly, our approach is applicable to wide range of objects, since most of the object contours can be approximated as smooth curves on the plane with elliptical fourier descriptors (EFD) [5]. And finally, it is a simple and automatic way to synthesize grasping points.

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[1]Control algorithm is implemented in Matlab software, which runs on a PC with intel i3 CPU @2.13 GHz and 4GB RAM.

The rest of this paper goes on as follows: Section II talks about the related work; Section III explains two-fingered frictional force-closure grasps; Section IV formulates and solves the two-fingered frictional force-closure grasp problem from a control point of view; Section V shows how to impose unilateral constraints for the grasp solution; Section VI illustrates the synthesis of force-closure grasping points on a deforming and moving object; Section VII proposes a general architecture to synchronize the synthesis of grasping points and reaching motion of the manipulator; and Section VIII concludes the paper and gives some future perspectives.

II. Related Work

Nguyen was the first who formulated and synthesized geometrically the two-fingered frictional force-closure grasps of polygonal objects [6]. Then, Faverjon and Ponce extended Nguyen's work for the case of piece-wise smooth curved objects [7]. Later, Blake solved the planar grasping problem of curved object from a symmetry theory point of view [8].

Today computer vision techniques are much skillful than a decade ago, and they can inform the robot about the object geometry [9], [10], [11]. Thus starting with a known object shape is not a strong assumption. In the literature, given the shape of the object, the synthesis of grasping points is performed through three groups of approaches: (i) *Brute Force* — This approach checks every contact configuration if it satisfies geometrical conditions associated with good grasps or not. It is computationally exhaustive; (ii) *Optimization* — This approach optimizes a grasp quality measure defined over the entire contact configuration space of the object (*e.g.*, [12], [13], [16]). It is much faster than a brut force approach; (iii) *Computational Geometry* — This approach first analyzes the shape of the object through geometric algorithms, and then computes directly the contact locations (*e.g.*, [7], [8]). It is more efficient and faster than the previous two types of approaches.

Apart from above, and compared to the rest of the vast literature of grasping, there are relatively few researchers who used control-based approaches (*e.g.*, [17], [18], [19]) to synthesize complete grasps. These works use multiple concurrent controllers (*e.g.*, learning based, behavior based, control bases) to overcome complex grasping problem by dividing it to smaller subtasks. These control-based approaches start with an approximate grasp and then modify it towards a good grasp by taking small contact steps along the unknown object surface based on local tactile feedback of fingertips.

Here, our work proposes a strong coherent complementary pre-stage to these control-based fine grasping approaches.

III. Two-Fingered Frictional Grasps

Here we explain two-fingered grasps with frictional point contacts. We choose a point contact with friction when friction exists between the fingertip and the object. In a point contact with Coulomb friction model, we can apply force to an object in any direction that is oriented within the friction-cone. The apex of the friction-cone coincides with the contact point, and the cone axis is aligned with the inward object boundary normal. The cone aperture is defined by the static friction coefficient $\mu \geqslant 0$, see Fig. 2.a. Now, if we imagine

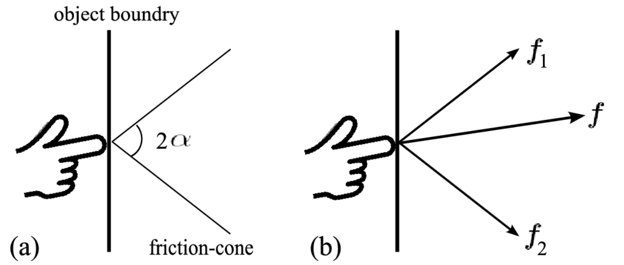

Fig. 2. Point contact with Coulomb friction model $\mu = \tan\alpha$ (a). Applied force f and the extreme generatrix forces $\{f_1, f_2\}$ of the friction cone (b).

the extreme rays of this friction-cone as generatrix forces, Fig. 2.b, then we can replace any applied force, which is pointing inside this friction-cone, with a positive combination of these generatrix forces:

$$f = af_1 + bf_2, \qquad a \geqslant 0, \;\; b \geqslant 0, \;\; f \geqslant \mathbf{0} \qquad (1)$$

where f and $\{f_1, f_2\}$ are the applied and the generatrix forces, respectively.

The necessary and sufficient condition for a force-closure grasp is to construct a torque-closure, since every translation is a rotation about an axis at infinity. For two-fingered frictional grasps, a torque-closure (thus also force-closure) grasp exists when each contact point lies within the friction-cone of the other [6]. See Fig. 3.

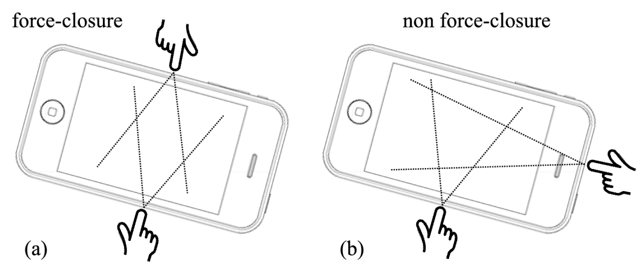

Fig. 3. Two-fingered frictional grasps of a cell phone. The grasp (a) satisfies a force-closure, but the grasp (b) does not.

IV. Grasp Problem from a Control Point of View

Let the contour of the object be a smooth (twice continuously differentiable), closed, simple (no self-intersections) curve, and let any contour point position be represented by a vector-valued function $p(s(t))$ where $s(t) \in I$ is an arc length parameter at time t over a set I. Then, the unit tangent $\underline{\tau}(s)$ and unit inward normal $\underline{n}(s)$ vectors of this curve at arc length s are given as follows:

$$\tau(s) = \frac{\partial p(s)}{\partial s} \triangleq \begin{bmatrix} \tau_x \\ \tau_y \end{bmatrix}, \quad \underline{\tau}(s) = \frac{\tau(s)}{\|\tau(s)\|} \qquad (2)$$

$$n(s) \triangleq \begin{bmatrix} -\tau_y \\ \tau_x \end{bmatrix}, \qquad \underline{n}(s) = \frac{n(s)}{\|n(s)\|} \qquad (3)$$

Afterwards, the control problem for a two-fingered frictional grasp can be stated as follows:

Control Problem: *Let $p(s_1^*)$ and $p(s_2^*)$ be the two unknown grasping points that satisfy the force-closure constraint ($FC : \Re^2 \times \Re^2 \to \Re$) on the object boundary*:

$$-\varepsilon \leqslant FC(\,p(s_1^*),\, p(s_2^*)\,) \leqslant \varepsilon \tag{4}$$

where ε is a positive threshold defined by the aperture of the friction-cone and it limits the force-closure region. Let also $p(s_1)$ and $p(s_2)$ be two arbitrary contact points which do not satisfy the force-closure constraint. Then find a control law $u(t)$ such that $(s_1(t) \to s_1^*, s_2(t) \to s_2^*)$ while $t_i > t_{i-1}$.

Now, we explain the "perfect" force-closure condition for two-fingered frictional grasp in geometrical terms.

Perfect Force-Closure: *This appears when the line joining the two contact points $\{\,p(s_1),\, p(s_2)\,\}$ aligns with both of the normals $\{\,\underline{n}(s_1), \underline{n}(s_2)\,\}$, or equivalently when the force closure constraint is $FC = 0$.*

From now on for the simplicity of notation, we will drop the arc length parameter s from the variables (*e.g.*, $p(s_1) \equiv p_1$, $\underline{n}(s_1) \equiv \underline{n}_1$, $\tau(s_1) \equiv \tau_1$). The perfect force-closure condition is satisfied when the following two differentiable error functions are minimized:

$$e_1 \,=\, \theta_1 \,=\, \arccos(\,\underline{\ell}_{12}^T\,\underline{n}_1\,) \tag{5}$$

$$e_2 \,=\, \theta_2 \,=\, \arccos(\,\underline{\ell}_{21}^T\,\underline{n}_2\,) \tag{6}$$

Note that these error functions are independent from the friction coefficient. When θ_1 (*resp.* θ_2) goes to zero, the unit vector $\underline{\ell}_{12}$ (*resp.* $\underline{\ell}_{21}$) pointing from the first (*resp.* second) contact point to the second (*resp.* first) aligns itself with the normal vector of the first (*resp.* second) contact point. If the friction coefficient is given, then we have chance to decide whether the contact points' configuration is force-closure or not before the errors are minimized completely. To do so, we first choose the biggest angle, $\theta = \max(\theta_1,\, \theta_2)$, and then compare it with the half aperture angle, α, of the friction-cone. If $\theta \leqslant \alpha$ then the contact points' configuration is force-closure, and as well as $m = (\alpha - \theta)/\alpha$ gives a precise quantitative measure for the stability of the grasp. Closer the m to 1, the more robust is the grasp. See Fig. 4.

We calculate the control law $u = [\dot{s}_1,\, \dot{s}_2]^T$ from Lyapunov's direct method. We first choose a positive definite Lyapunov function $V_\theta = 1/2\,(e_1^2 + e_2^2) > 0$. Control law u is hidden in $\dot{V}_\theta$, and it should be chosen such that $\dot{V}_\theta < 0$ so as to move contact points toward a force-closure configuration:

$$\dot{V}_\theta \,=\, e_1\,\dot{e}_1 + e_2\,\dot{e}_2 \,=\, e_1 A_1\,u + e_2 A_2\,u < 0 \tag{7}$$

From (7), we can write the following control law:

$$u \,=\, -\lambda_\theta\,(A_1^T\,e_1 + A_2^T\,e_2) \tag{8}$$

where λ_θ is a positive scalar controller gain. See Appendix for matrices A_1 and A_2. The control law u moves the contact points to the closest solution. Now, we can update contact points' positions with the control law $u = [u_1, u_2]^T$ as below:

$$p(s_1) \,=\, p(s_1 + \Delta t\,u_1)\,, \qquad p(s_2) \,=\, p(s_2 + \Delta t\,u_2) \tag{9}$$

where $\Delta t \,=\, t_i - t_{i-1}$ is the control sampling time.

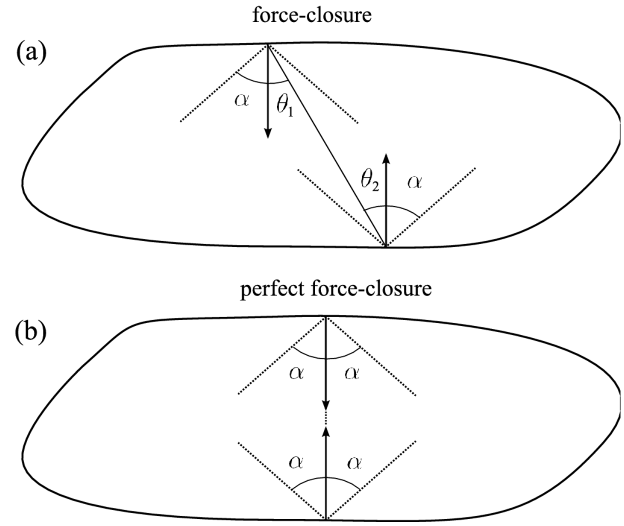

Fig. 4. Force-closure $0 < \theta_1 \leqslant \alpha$ and $0 < \theta_2 \leqslant \alpha$ (a). Perfect force-closure $\theta_1 = \theta_2 = 0$ (b).

Example: In Fig. 5.a, we see an initial non force-closure configuration of the contact points on a cell phone with the normal vectors. In Fig. 5.b, we see the final force-closure configuration of the contact points with their trajectories on the cell phone boundary. In Fig. 5.c, we see the errors of the angles θ_1 and θ_2. Assuming that the friction between a fingertip and the cell phone is $\mu = 0.364$ ($\alpha = 20°$), we can conclude that the configuration is a force-closure, when both errors of the angles are smaller than the half-angle α of the friction cone.

V. Control Law with Unilateral Constraints

Object can be larger in one dimension than the maximal opening of any two fingers. In this case, we do not want a solution for grasping points that stays at the extremities of the object. We therefore integrate a maximal opening distance d_{max} of fingers in the synthesis of grasping points as a unilateral constraint such that:

$$c \,=\, \|p_1 \,-\, p_2\| < d_{max} \tag{10}$$

Constraint (10) lets us to check if the solution is in the workspace of fingers ($c < d_{max}$) or not ($c \geqslant d_{max}$). For the new control law, we add a constraint error function which attracts contact points to the dexterous workspace of fingers:

$$e_d \,=\, \kappa\,(1/2)\,\big((p_1 - p_2)^T\,(p_1 - p_2) - d^2\big) \tag{11}$$

where d is, for example, half-distance of the maximal opening of the fingers; and where κ is defined as follows:

$$\begin{array}{llll} if \;\; c \geqslant d_{max} & then & \kappa = 1 & until \;\; e_d \leqslant 0 \\ \;\; otherwise & & \kappa = 0 & \end{array} \tag{12}$$

We choose another positive definite Lyapunov function for the constraint $V_d \,=\, (1/2)\,e_d^2 > 0$. The time derivative of Lyapunov constraint function should be negative definite so that the constraint error decreases:

$$\dot{V}_d \,=\, e_d\,\dot{e}_d < 0 \tag{13}$$

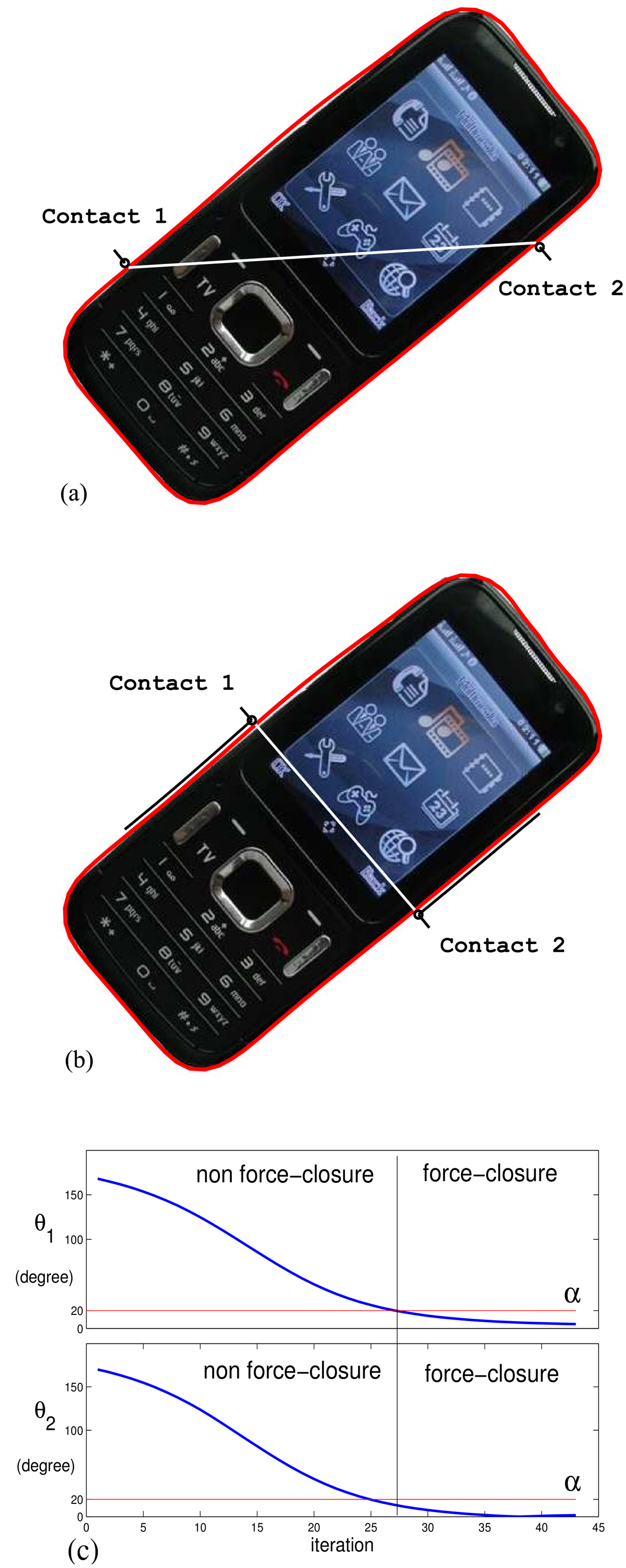

Fig. 5. Initial non force-closure configuration (a). Final force-closure configuration with stability measure $m = 0.95$ (b). Evolution of angle errors during alignment of the joining line with the normal vectors of the contact points (c).

where

$$\dot{e}_d = D u, \qquad D = [(p_1 - p_2)^T \tau_1, \; -(p_1 - p_2)^T \tau_2] \tag{14}$$

Then, we write together (7) and (13) to develop the new control law:

$$\dot{V}_\theta + \dot{V}_d < 0 \tag{15}$$

which gives:

$$e_1 A_1 u + e_2 A_2 u + e_d D u < 0 \tag{16}$$

From (16) the new control law, which satisfies (15), can be written as below:

$$u = -\Lambda_1 (A_1^T e_1 + A_2^T e_2) - \Lambda_2 D^T e_d \tag{17}$$

where Λ_1 and Λ_2 are 2×2 positive definite diagonal controller gain matrices. Before using this control law, we do a final trick to normalize the contributions from each of the errors, since the force-closure errors are in angles and the constraint error is in squared distance. Otherwise, we can suffer a little bit for a proper choice of the controller gains Λ_1 and Λ_2. Without loss of generality, the controller gains Λ_1 and Λ_2 can scale the values but cannot change their signs. Therefore, we can rewrite the control law (17) as follows:

$$u = -\lambda_\theta \tanh(A_1^T e_1 + A_2^T e_2) - \lambda_d \tanh(D^T e_d) \tag{18}$$

where λ_θ and λ_d are the new positive scalar controller gains, and now they can be easily chosen. Arranging them such that $\lambda_\theta < \lambda_d$ gives more priority to the constraint, and this forces the solution to be in the workspace of the fingers. This new control law can be thought as a sliding mode control (SMC), which is a robust control [14]. To increase the contact stability [15], one can add more constraints such that the solution: (i) has smaller distance between the contact points; (ii) has lower curvature around the contact points; and (iii) is closer to the center of mass of the object.

We can generalize (18) for multi-fingered grasps with one force-closure and k constraint error functions:

$$u = -\left(\lambda_{FC} \tanh\left(\frac{\partial \dot{V}_{FC}}{\partial u}\right) + \sum_{i=1}^{k} \lambda_{ci} \tanh\left(\frac{\partial \dot{V}_{ci}}{\partial u}\right) \right) \tag{19}$$

where $\dot{V}_{FC}$ and $\dot{V}_{ci}$ are time derivatives of the designed positive definite Lyapunov functions of the force closure and the constraints, respectively. Assuming that the priorities of constraints are ordered from the lowest to the highest, the controller gains can be written as follows:

$$0 < \lambda_{FC} < \lambda_{c1} \leqslant \lambda_{c2} \leqslant \ldots \leqslant \lambda_{ck} \tag{20}$$

The priority order of the constraints depends on the object, the hand, and the task. It should thus be done carefully. The force-closure controller gain λ_{FC} should have the lowest priority so that the solution satisfies the constraints, or we can disable it until all the constraints are satisfied:

$$\lambda_{FC} = \lambda_{FC} \left(1 - (\kappa_{c1} | \kappa_{c2} | \ldots | \kappa_{ck})\right) \tag{21}$$

where $\kappa_{ci} \in \{0, 1\}$ are the constraints' on/off values, and where $|$ is an OR logic gate.

Example: In Fig. 6.a, we see a pen with its boundary curve and its initial non force-closure contact points. In Fig. 6.b, we see the synthesis of force-closure grasping points without the maximal distance constraint, and in Fig. 6.c, we see this with the maximal distance constraint. The initial contact point locations in two cases are the same.

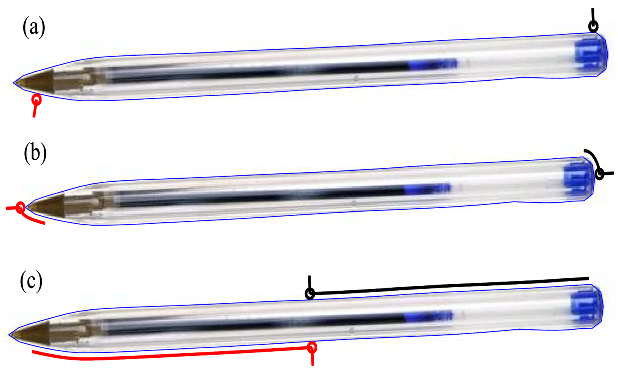

Fig. 6. Synthesis of grasping points on a pen. The boundary curve of the pen and the initial contact points (a). Synthesis of a force-closure configuration without the constraint (b), and with the constraint (c).

VI. Robustness to Shape and Pose Changes

Arc-length positions s_1 and s_2 of synthesized force-closure grasping points are intrinsically invariant under uniform scaling and translation, but not under rotation. Here, we show that the control law is also robust to rotations and to deformations. We illustrate this on a circle curve which deforms itself continuously to an ellipse. We also at the same time rotate the deforming curve counter-clockwise. Let the initial circle has a radius r units (*i.e.*, an ellipse with major and minor lengths equal to radius, $a = b = r$). Let also the deformation of the ellipse be given by $a = a + \Delta t\, v_a$ and $b = b + \Delta t\, v_b$, where v_a and v_b are the deformation velocities along the major and minor axes at each sampling time. Finally, let the initial rotation angle β be changing with an angular velocity ω_β such that $\beta = \beta + \Delta t\, \omega_\beta$. Figure 7 shows the tracking of a force-closure configuration on the curve while the curve deforms and rotates. We conclude this part by stating that the control law is robust to small Euclidean disturbances on the pose and also to the small deformations on the shape of the object. Or from another point of view, the control law can track, without any extra computational effort, a force-closure configuration on the object even though the object moves and deforms.

VII. Synchronizing Grasping Points Synthesis and Reaching Motion of the Manipulator

Here we give a general architecture and formulation to synchronize the synthesis of grasping points and the reaching motion of the manipulator. See Fig. 8.

Let $\mathbf{f}_1$ and $\mathbf{f}_2$ be the Cartesian positions of the two fingertips with respect to manipulator base frame. Then, we relate the velocities of the fingertips to the velocities of the joints of the manipulator through the Jacobians J_1 and J_2:

$$\dot{\mathbf{f}}_1 = J_1\,\dot{\mathbf{q}}, \qquad \dot{\mathbf{f}}_2 = J_2\,\dot{\mathbf{q}} \tag{22}$$

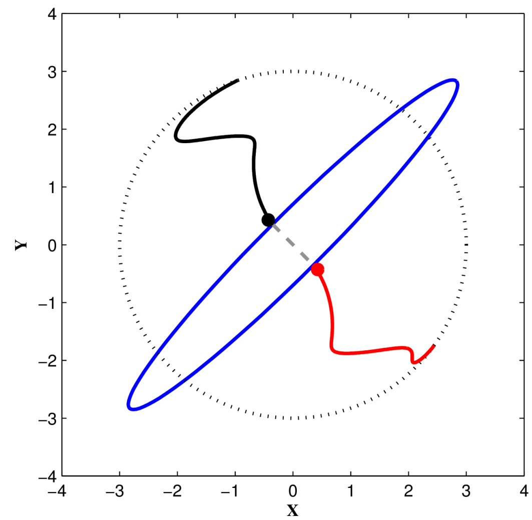

Fig. 7. Control law keeps the grasping points in the force-closure configuration while the curve deforms and rotates. Dots show the initial circle curve with $r = 3$. Deformations on the major and minor axes are $a = [3 \to 4]$ and $b = [3 \to 0.5]$, and changes in the rotation angle is $\beta = [0° \to 45°]$. Blue ellipse is the final deformed and rotated curve with its force-closure grasping points (black and red solid big dots). Trajectories of the force-closure grasping points are shown in red and black solid line traces.

where $\mathbf{q} = [\mathbf{q}_{arm}^T, \mathbf{q}_{finger1}^T, \mathbf{q}_{finger2}^T]^T$ is the vector of joint coordinates. Let also $\mathbf{p}_1^*$ and $\mathbf{p}_2^*$ be the desired positions of the fingertips computed from the contact point locations which are coming from the control-based synthesis algorithm. Then, we can build a simple tracking error for fingertips:

$$\mathbf{e}_1 = \mathbf{f}_1 - \mathbf{p}_1^*, \qquad \mathbf{e}_2 = \mathbf{f}_2 - \mathbf{p}_2^* \tag{23}$$

We follow again Lyapunov's direct method to write the control law of the manipulator during reaching motion. We choose the following Lyapunov function for the manipulator:

$$V_s = \frac{1}{2}\left(\mathbf{e}_1^T\,\mathbf{e}_1 + \mathbf{e}_2^T\,\mathbf{e}_2\right) > 0 \tag{24}$$

and differentiating this function with respect to time yields:

$$\dot{V}_s = \mathbf{e}_1^T\,\dot{\mathbf{e}}_1 + \mathbf{e}_2^T\,\dot{\mathbf{e}}_2 < 0 \tag{25}$$

which should be smaller than zero at each iteration so that the manipulator pursues the trajectories of contact points during

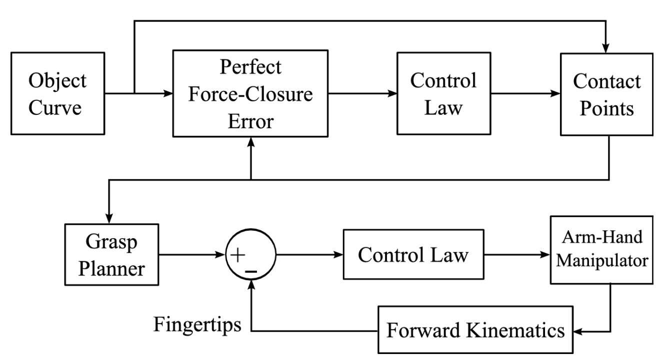

Fig. 8. Synchronizing synthesis of grasping points with reaching motion.

synthesis. We rewrite (25) explicitly to appear the control law $\dot{\mathbf{q}}$ of the manipulator as below:

$$\mathbf{e}_1^T\,(J_1\,\dot{\mathbf{q}} - \dot{\mathbf{p}}_1^*) + \mathbf{e}_2^T\,(J_2\,\dot{\mathbf{q}} - \dot{\mathbf{p}}_2^*) \,<\, 0 \tag{26}$$

From (26), we calculate the control law as follows:

$$\dot{\mathbf{q}} = -\lambda\,A^T + \frac{A^T B}{A A^T}\,, \qquad \lambda > 0 \tag{27}$$

where A and B are as below:

$$A = \mathbf{e}_1^T J_1 + \mathbf{e}_2^T J_2\,, \qquad B = \mathbf{e}_1^T\,\dot{\mathbf{p}}_1^* + \mathbf{e}_2^T\,\dot{\mathbf{p}}_2^* \tag{28}$$

VIII. CONCLUSIONS

In this paper, we reformulated the two-fingered frictional planar force-closure grasping problem from a control point of view. Strong sides of this approach are: (i) It is independent from the friction coefficient; (ii) It yields a quantitative measure for the final force-closure configuration; (iii) It proposes a general control law for multi-fingered grasps with unilateral constraints; (iv) It is robust to small changes in shape and pose of the object; (v) It tracks a force closure configuration on the object even though the object moves and deforms continuously; and (vi) It can be synchronized with the reaching motion of the arm-hand manipulator.

Weak side of this approach is: On a concave object if the initial contact points are far from a force closure solution, then the contact points can stick to a local minima which is not a force closure. Proposed approach guarantees a force-closure solution for the time being on the convex objects.

Future works are: (i) to guarantee a feasible force-closure solution on the concave objects too; (ii) to study two-fingered frictional force-closure planar grasps of deformable objects; and (iii) to define differentiable force-closure error functions for multi-fingered frictional force-closure grasps.

APPENDIX

In order to relate force-closure angle θ_1 in (5) to the control law u, we differentiate it with respect to time:

$$\dot{\theta}_1 = k_1\,\frac{d}{dt}(\underline{\ell}_{12}^T\,\underline{n}_1)\,, \qquad k_1 = -\frac{1}{\sqrt{1-(\underline{\ell}_{12}^T\,\underline{n}_1)^2}} \tag{29}$$

where $\underline{\ell}_{12} = (p_2 - p_1)/\|p_2 - p_1\|$, and where

$$\frac{d}{dt}(\underline{\ell}_{12}^T\,\underline{n}_1) = \underline{n}_1^T\,M_{\ell 12}\,(\dot{p}_2 - \dot{p}_1) + \underline{\ell}_{12}^T M_{n1}\,\dot{n}_1 \tag{30}$$

The matrices $M_{\ell 12} \in \Re^{2\times 2}$ and $M_{n1} \in \Re^{2\times 2}$ are as follows:

$$M_{\ell 12} = \frac{1}{\|p_2 - p_1\|}\,(I_2 - \underline{\ell}_{12}\,\underline{\ell}_{12}^T)\,,\;\; M_{n1} = \frac{1}{\|n_1\|}\,(I_2 - \underline{n}_1\,\underline{n}_1^T) \tag{31}$$

and where I_2 is a 2×2 identity matrix. Using equations (29), (30), and (31), we can write the velocity of the angle θ_1 in terms of the control law $\dot{\theta}_1 = A_1\,u$ where $A_1 \in \Re^{1\times 2}$ is as follows:

$$A_1 = \left[k_1\,(\underline{\ell}_{12}^T M_{n1}\,\frac{\partial\, n_1}{\partial\, s_1}) - k_1\,(\underline{n}_1^T\,M_{\ell 12}\,\tau_1)\,,\; k_1\,(\underline{n}_1^T\,M_{\ell 12}\,\tau_2)\right] \tag{32}$$

where τ_1 and τ_2 are the tangent vectors of the contact points. Similarly to the above equations, we can relate θ_2 to the control law as $\dot{\theta}_2 = A_2\,u$ where $A_2 \in \Re^{1\times 2}$ is as follows:

$$A_2 = \left[k_2\,(\underline{n}_2^T\,M_{\ell 21}\,\tau_1)\,,\; k_2\,(\underline{\ell}_{21}^T\,M_{n2}\,\frac{\partial\, n_2}{\partial\, s_2}) - k_2\,(\underline{n}_2^T\,M_{\ell 21}\,\tau_2)\right] \tag{33}$$

$$k_2 = -\frac{1}{\sqrt{1-(\underline{\ell}_{21}^T\,\underline{n}_2)^2}}\,, \qquad \underline{\ell}_{21} = (p_1 - p_2)/\|p_1 - p_2\| \tag{34}$$

$$M_{\ell 21} = \frac{1}{\|p_1 - p_2\|}\,(I_2 - \underline{\ell}_{21}\,\underline{\ell}_{21}^T)\,,\;\; M_{n2} = \frac{1}{\|n_2\|}\,(I_2 - \underline{n}_2\,\underline{n}_2^T) \tag{35}$$

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